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2. Recall that many bosons (Cooper pairs) may occupy the ground state.

Identify superconducting current with that carried by Cooper pairs.

Normal current carried by single electrons (fermions) in excited states.

Ground state separated from excited states by energy gap  $\Delta$ .

Pairs have range of interaction - called coherence length ( $\sim 10^{-6} \text{ m}$ )

3. Q. Is the superconducting current a Bose-Einstein condensation of Cooper pairs?

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A. Large numbers of Cooper pairs can occupy the ground state and cause Superconducting current

Wavefunction of pairs is correlated -  
single wlf for all pairs as in  
Bose-Einstein condensate

But evaluation of Bose-Einstein  
condensation temperature  $T_B$  using  
actual electron density  $\left(\frac{N}{V}\right)$  gives

$$T_B \gg T_c$$

Thus:

(i) Superconductivity depends on a  
condensation of bosons (Cooper pairs)  
into ground state

- (ii) Critical temperature  $T_c$  set by strength of pairing interaction not by Bose - Einstein condensation

Further points.

4. Magnetic field acts to destroy Superconductivity because its action is to align electron spins  $\parallel B$  - tending to break up  $\uparrow\downarrow$  pairs
5. BCS theory relates energy gap  $\Delta(0)$  and electron-lattice interaction strength  $V$  via

$$\Delta(0) = 1.76 kT_c = 2\hbar\omega_D \exp\left(-\frac{1}{g(\mu) \cdot V}\right)$$

where  $\omega_D = \frac{k\theta_D}{\hbar}$  ← Debye temperature  
and  $g(\mu)$  is density of states at Fermi surface

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6. Isotope effect — first evidence that lattice concerned in pairing interaction

For Superconductors Hg, Pb, Sn, Zn ..

Value of  $T_c$  for samples of different isotopes of same element related to isotope mass  $M$  by

$$T_c \propto \left( \frac{1}{M} \right)^{1/2}$$

Follows from  $T_c \propto \omega_D \propto \left( \frac{k}{M} \right)^{1/2}$

where  $k$  is bond constant.

$M$  is isotopic mass.

7. Experiment shows energy gap  $\Delta$  effectively constant over range

$$0 < T < T_c/2$$

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For temperature range  $\frac{T_c}{2} < T < T_c$   
approximate relation

$$\Delta(T) \approx 3.1 \cdot k T_c \left[ 1 - \frac{T}{T_c} \right]^{\frac{1}{2}}$$

— see sheet.

8. Seen above that —

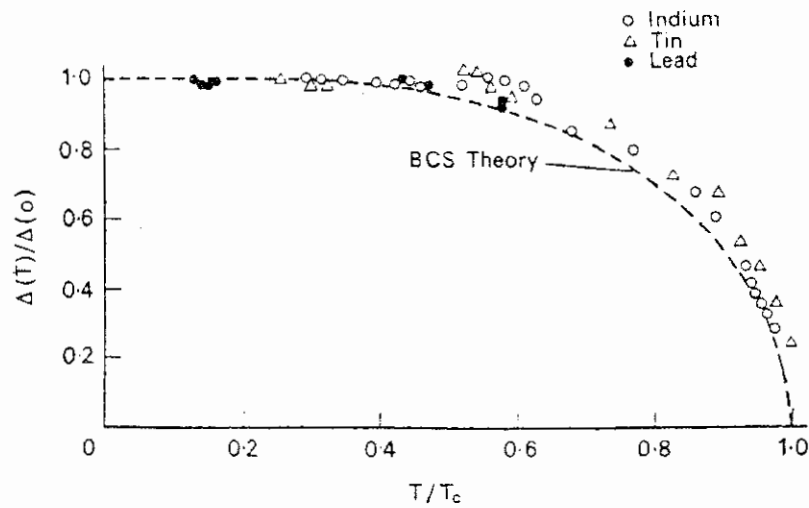
Large electron lattice interaction  $V$  related  
to high  $T_c$

In normal state large  $V$  means  
large (for metal) resistivity

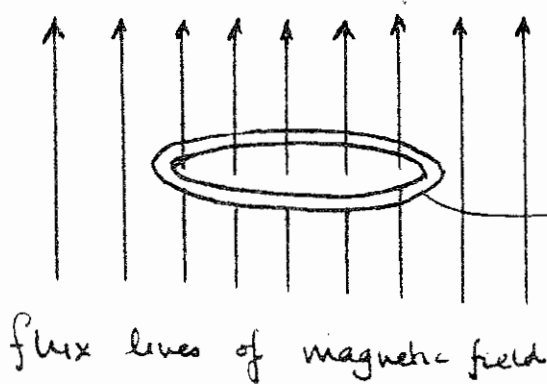
Connection between superconductivity  
and poor metallic conductivity in  
normal state

Reason why good conductors Cu, Ag,  
Au ... do not show  
superconductivity.

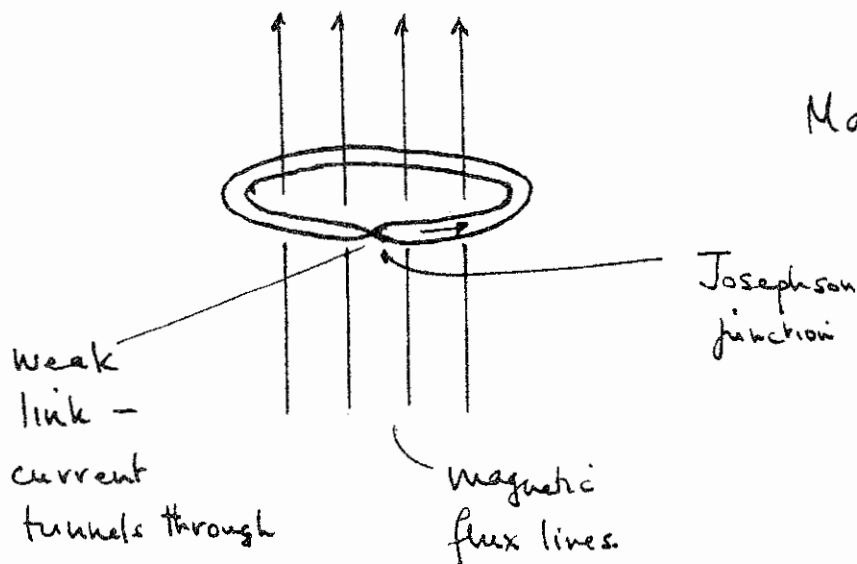
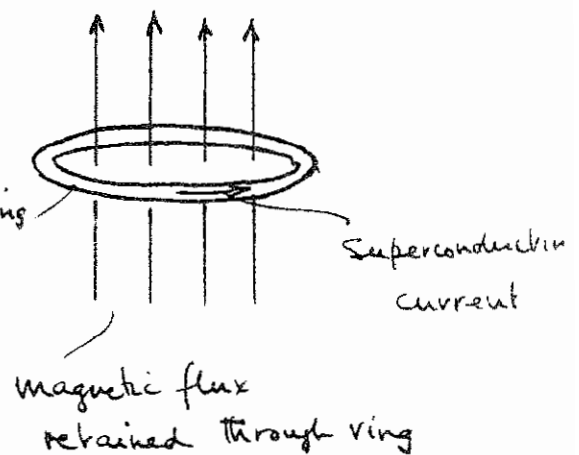
Variation of energy gap  $\Delta(T)$  with temperature.



Magnetic field on



Field turned off



$$\text{Magnetic field } B = \frac{\text{Flux } \phi}{\text{Area } A.}$$

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Flux quantisation through a superconducting ring

Schematic diagrams — see sheet.

When magnetic field removed — magnetic flux passing through hole in the ring is maintained by superconducting current flowing around ring.

Can be shown that total flux through ring  $\phi$  is

$$\phi = n \phi_0$$

where  $n = \text{integer}$

$$\phi_0 = \text{unit of flux} = \frac{h}{2e} \text{ Tm}^2$$

Create weak link in ring — Josephson junction — flux can escape — one line at a time — see sheet.

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By inductive coupling of coils to Josephson loop — get signal for each  $\phi_0$  that enters or escapes from loop.

Basis of SQUID

(Superconducting quantum interference device)

— most sensitive method of measuring magnetic field.

BCS prediction

$$\phi_0 = \frac{h}{q}$$

where  $q$  is charge of current carrier

Measured value  $\phi_0 = \frac{h}{2e}$

gives  $q = 2e$

confirms Superconducting current carried by electron pairs.



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Type I and Type II Superconductors.

Different  $M$  vs  $H$  graphs - see sheet.

Type I - for  $0 < H < H_c$  all flux  
excluded from superconducting cylinder

Type II - for  $0 < H < H_{c1}$  same as Type I

for  $H_{c1} < H < H_{c2}$  flux line  
penetration (mixed state)

for  $H_{c2} < H$  normal state

Penetrating flux quanta in mixed state

minimise energy by equalising their

Separation - creates flux line lattice

- see sheet.

In practice impurities in sample can pin  
flux lines

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Type II Superconductors more practical in high field uses - magnet coils - since usually  $H_{c2} > \text{Type I } H_c$ .

Parallel between flux line penetration of Type II Superconductor and quantised vortices (rotons) in liquid  $He^+ II$ .

New Superconducting materials.

For elements highest  $T_c = 9.5 K$  Nb.

For alloys "  $T_c = 23.2 K$   $Nb_3 Ge$

Many attempts to find superconductor with higher  $T_c$  (and  $H_c$ ).

In 1987 - new type of oxide (ceramic)

Superconductors found with higher values of  $T_c$

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Values	$T_c$
$\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$	36 K
$\text{YBa}_2\text{Cu}_3\text{O}_7$	90 K
$\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$	120 K

Materials have crystal structure of Cu-O planes spaced apart by other atoms  
- see sheet.

Superconductivity occurs mainly along Cu-O planes

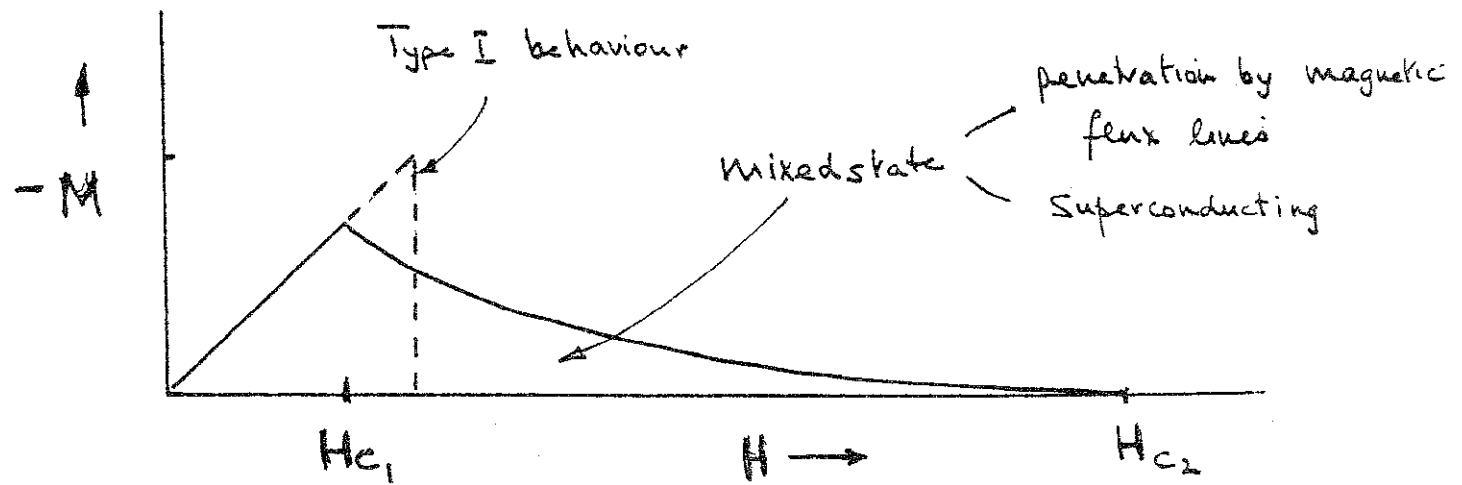
High  $T_c$  means strong pairing interaction

Mechanism for this still not clear

Materials are type II with large  $H_{c2}$

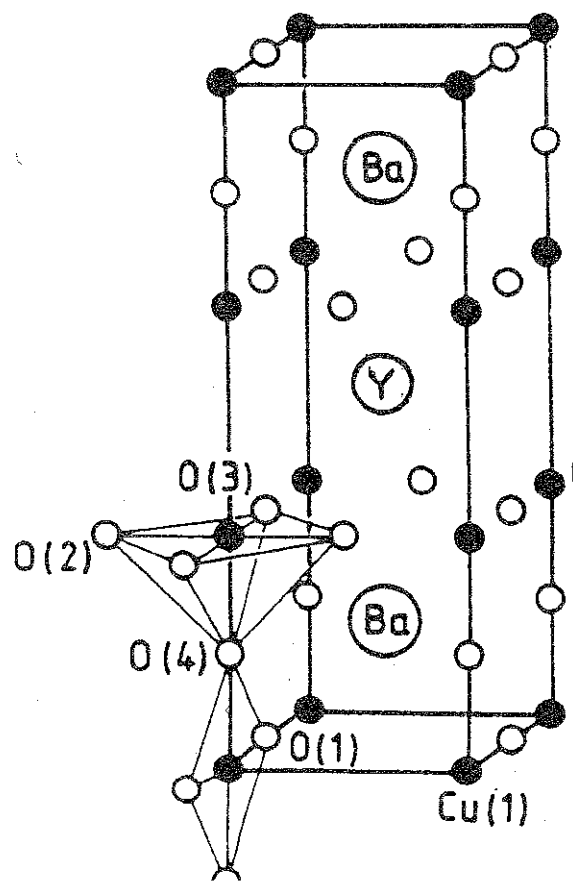
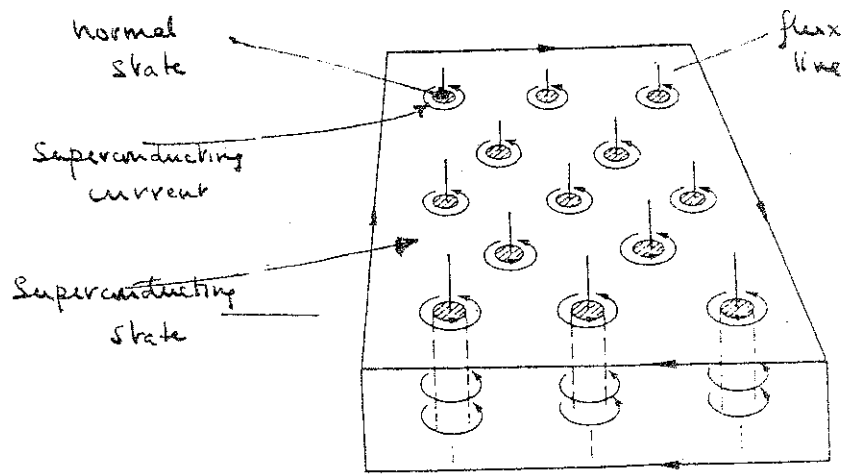
but so far have been unable to pass large currents without becoming normal.

# Type II Superconductors



Picture of mixed state of Type II Superconductor.

Note lattice (pattern) of flux lines



Crystal structure of  $YBa_2Cu_3O_7$  showing  $Cu-O$  planes

←  $Cu-O$  plane

←  $Cu-O$  plane

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## Superfluid $\text{He}^3$

Seen that Superconducting behaviour arises from Bose-Einstein condensation where bosons are coupled pairs of electrons

Question. Can  $\text{He}^3$  atoms (fermions) pair to form bosons allowing a

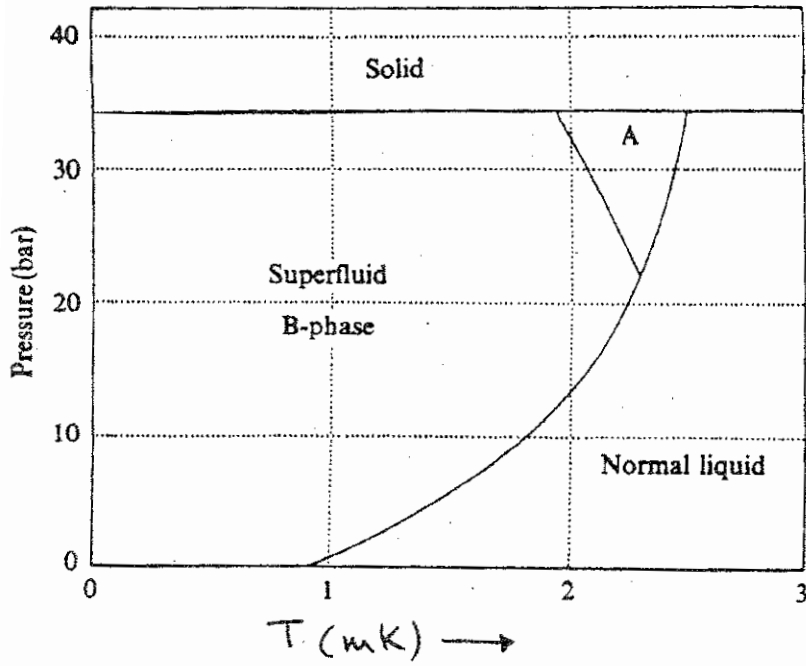
Bose-Einstein condensation and superfluid properties ?

Ans. Yes - but at low temps  $\lesssim 2\text{mK}$ .

Low temperature phase diagram of  $\text{He}^3$

— See sheet.

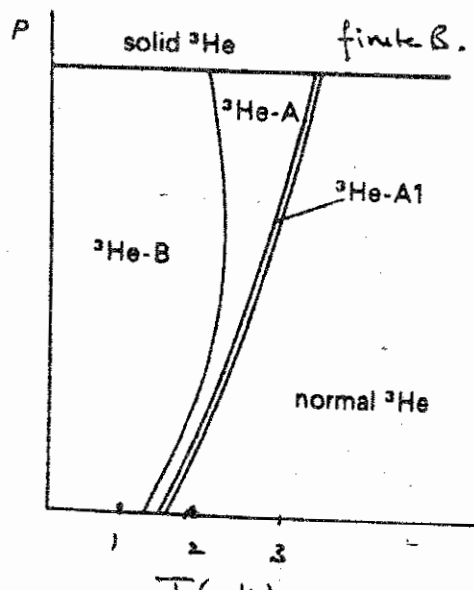
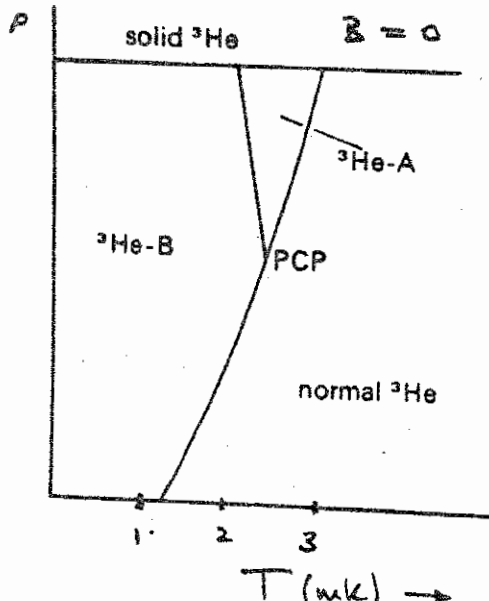
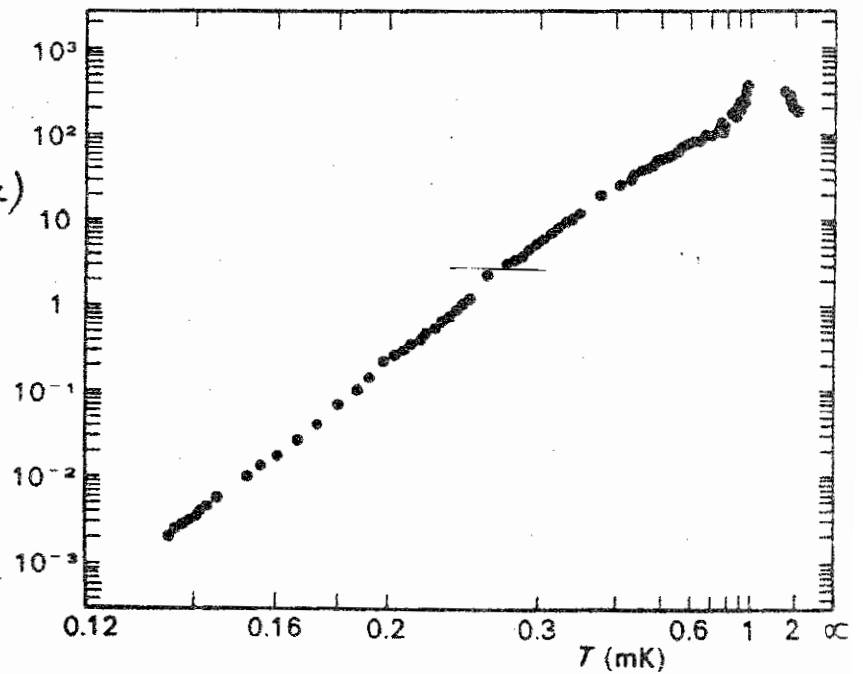
# Superfluid $^3\text{He}$



Low temperature phase diagram

Damping of oscillating wire in liquid  $^3\text{He}$

$\Delta\nu$  (Hz)



Effect of magnetic field on phase diagram

Diagram shows for  $T \leq 2\text{mK}$  two different Superfluid phases - phase A  
phase B.

Properties of these phases complex - just discuss main points.

Points.

1. Proof of superfluid nature.

Expt - damping of wire vibrating at freq  $\nu$  in liquid  $\text{He}^3$ .

Stimulate vibration of wire - resonance at natural freq  $\nu$

Measure width of resonance  $\Delta\nu$

$\Delta\nu$  depends on damping force (viscosity) of liquid  $\text{He}^3$

Plot of  $\Delta v$  versus  $T$  — see sheet.

Damping force decreases by  $10^5$  between  
1.04 mK and 0.14 mK.

2. Viscosity expts (like above) consistent  
with 2 fluid model where  
fluid normal at critical temp  $T_c$   
but proportion of superfluid increases as  
temp decreased

All superfluid as  $T \rightarrow 0$  K.

Superfluid seen as  $He^3$  pairs in g/state

Normal fluid "  $He^3$  single atoms in  
excited states

As in superconductors — strength of pairing  
interaction determines critical temp  $T_c$



3. Mechanism of pairing. — not phonons  
but type of magnetic interaction.

Atom 1 polarises surrounding  $\text{He}^2$  atoms

Atom 2 interacts with polarisation trail.

Details complex.

Interaction weak — since  $T_c \sim 2\text{mK}$ .

4. Character of pairs

In Superconductors — electron pairs have

$$L = 0, S = 0$$

Superfluid  $\text{He}^2$  pairs shown (by expt)

to have  $S = 1$

Then Pauli principle (total wff of pair  
antisymmetric with  
particle exchange)

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actually  $L = 1$ .

Thus  $\text{He}^3$  superfluid pairs have  $L=1, S=1$ .

From quantum mechanics.

$S = 1$  has substates

$$S_z = +1 \quad \uparrow\uparrow$$

$$S_z = 0 \quad \frac{1}{\sqrt{2}} \{ \uparrow\downarrow + \downarrow\uparrow \}$$

$$S_z = -1 \quad \downarrow\downarrow$$

Distinction between

Phase A contains  $S_z = +1 \quad \uparrow\uparrow$

and  $S_z = -1 \quad \downarrow\downarrow$

pairs only

Phase B contains  $S_z = +1, 0, -1$  pairs

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5. Effect of magnetic field.

Pairs with  $S = 1$   $S_z = +1$  } have  
 $S = 1$   $S_z = -1$  } magnetic  
moment

— thus superfluid affected by magnetic field.

Effect of phase diagram on field — see sheet.

Phase A more magnetic than Phase B —

Since all pairs have magnetic moment

For field  $> 0.6 \text{ T}$  — no phase B.

New phase  $A_1$  occurs between normal  
fluid and Phase A.

Phase  $A_1$  thought to contain just

$S_z = +1$   $\uparrow \uparrow$

or  $S_z = -1$   $\downarrow \downarrow$  pairs

6. Pairs have  $\underline{L}$  vector normal to  $\underline{S}$   
to minimise pair energy.

In phase A

$\underline{L}$  vector wants to be (i)  $\perp$  to walls of container

(ii)  $\perp$  to  $\underline{S}$

While in magnetic field  $\underline{S}$  wants to be  $\parallel$  to field.

These requirements usually conflict giving rise to complex, anisotropic effects.

Phase B similarly complex.

Further details

Introduction to Liquid Helium

Wilks and Betts.

Main point.

Superfluid  $\text{He}^3$  exists by pairing

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of  $\text{He}^3$  atoms to form bosons that  
make Bose-Einstein condensation to ground  
state.

Practical methods of reaching low temperature

Energy  $U$ , Entropy  $S$  and Temperature  $T$ .

At low temperatures it is more helpful to associate temperature  $T$  with entropy rather than energy.

Recall Entropy  $S = k \ln \Omega$

where  $\Omega$  = number of microstates of system

Qualitatively  $S$  represents the disorder in the System

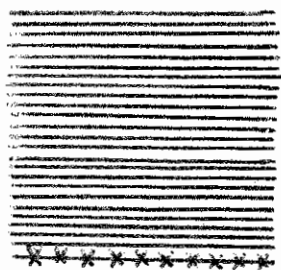
- in an equilibrium system as  $T \rightarrow 0$   
 system goes into ground state  $\Omega = 1$   
 and  $S = 0$   
 but energy  $U$  is not necessarily  $= 0$ .

## I Illustrations

- (i) System of
- $N$
- bosons.

As  $T \rightarrow 0$

Bosons condense into ground state



giving

$U \rightarrow 0$

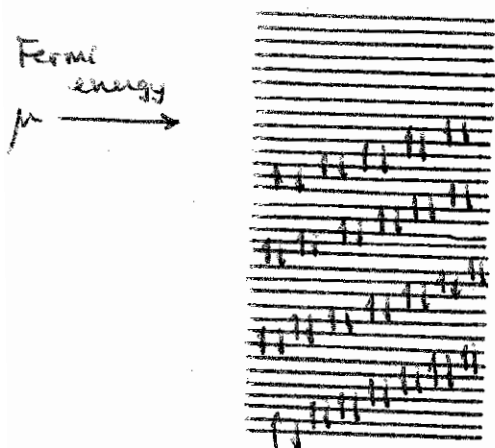
(not counting quantum zero point energy)

$S \rightarrow 0$

- (ii) System of
- $N$
- electrons (fermions)

As  $T \rightarrow 0$

Electrons condense to lowest set of states - but Pauli principle does not allow two electrons in same quantum state



As  $T \rightarrow 0$

Energy  $U \rightarrow \frac{3}{5} N \mu$

$S \rightarrow 0$